

Characterizing the Cognitive Demands of Mathematical Tasks

A Task-Sorting Activity

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GOALS

- Raise awareness of how mathematical tasks differ with respect to their levels of cognitive demand
- Highlight the importance of analyzing and discussing tasks in order to determine the level of thinking required to solve them

The point is not that one type of task is better than another; rather, it is important to know the potential of a task so that it can be appropriately mapped on to the goals for students' learning. This activity also raises teachers' awareness of "worthwhile mathematical tasks" as defined by *Professional Standards for Teaching Mathematics* (NCTM 1991).

CONTENT OF THE TASKS TO BE SORTED

- Number and operation
- Algebra
- Data analysis
- Geometry
- Measurement

GRADE LEVEL: K–12

TIME: 75–90 minutes

MATERIALS

- Copies of Martha's Carpeting task (transparency 6.1) and the Fencing task (transparency 6.2)
- Tools for working on the two tasks (e.g., calculators, grid paper, square tiles)
- Copies of the sixteen-task card sort at the appropriate grade level (i.e., 3–5, 6–8, 9–12)
- Copies of the recording sheet
- Overhead transparency of the recording sheet
- Overhead transparencies of each task
- Chart paper

IMPLEMENTATION

Arbaugh and Brown (2004) provide background information on what this task-sorting activity accomplishes and ways that teachers respond when they are asked to complete it.

Problem Solving (15–20 minutes)

Begin by having teachers work individually on Martha's Carpeting and the Fencing tasks for five minutes and then continue to work on the tasks

with a partner. This gives teachers both a first-hand experience in solving a high-level (Fencing task) and low-level (Martha's Carpeting task) task and a basis for distinguishing between the levels of tasks during the sorting activity.

Orchestrate a whole-group discussion to allow teachers to share various strategies for solving the tasks. Teachers can then discuss the similarities and differences between the two tasks. For example, teachers often note that although both tasks pertain to the same mathematical content (area of a rectangle), they require different types of thinking from the students (with one requiring the application of a formula and the other requiring a deeper level of thinking). The discussion should make salient the fact that all tasks are not created equal—different tasks require different levels and kinds of thinking from students. Tasks that require students to perform a memorized procedure in a routine manner (such as Martha's Carpeting task) lead to one type of opportunity for students' thinking. These are referred to as low-level tasks. Tasks that demand engagement with concepts and that stimulate students to make purposeful connections to meaning or relevant mathematical ideas (such as the Fencing task) lead to a different set of opportunities for students' thinking. These are referred to as high-level tasks.

Sorting Activity (30 minutes)

Teachers are now ready to begin work on the sorting activity. Working in pairs or triads, have teachers sort the cards into two groups—those that they consider to be high level, and those they consider to be low level—and develop the criteria for each category. As groups finish sorting, have them record their decisions on an overhead. (This way you will be able to see how each group categorizes each task.)

Discussion of Sorting Activity (30–45 minutes)

Note: You may find it helpful to read Smith and Stein (1998) prior to facilitating the sorting activity so that you are familiar with the characteristics of tasks at both levels.

Orchestrate a whole-group discussion of the sort. Begin by focusing on a task that the majority of the groups characterized as low level. Ask teachers to describe the characteristics of this task while you record their statements on chart paper. Move on to consideration of other tasks that were characterized as low level, adding to and clarifying the criteria as needed. You should then repeat this process for

high-level tasks.

Once the group has come to some agreement regarding a subset of tasks at each level, you are ready to focus on the tasks for which there is some disagreement. For each task, ask teachers whether it is more like the high-level or the low-level tasks, comparing the characteristics on your list. If teachers indicate that the task is low level, ask questions such as "What is the rule or procedure you would use to solve the task?" or "What have you memorized that you are being asked to recall?" If teachers indicated that the task is high level, ask questions such as "What is it you have to think about in order to solve the task?" or "What decisions or judgments do you have to make?"

Participants' Anticipated Responses

Results from the entire group will most likely show that the teachers disagree about the placement of a number of tasks. Those differences should prompt rich discussions with regard to analysis and characteristics of different levels of tasks. Disagreements often result from making assumptions regarding the cognitive level of a task based on surface features of the task or from equating "high level" with "difficult." For example, some teachers might assume that the feature "requires an explanation" is always associated with tasks with high-level demands. Although many tasks in the sorts are consistent with this view, others can serve as counterexamples to this assertion. For example, in the middle school sort task A (a high-level task) and task N (a low-level task), both require an explanation. The point is to encourage teachers to dig beneath the surface in determining the level of thinking required to complete a task. The following questions should foster lively discussion about these issues.

1. Does a particular feature (e.g., writing an explanation as part of your answer, drawing a picture to explain what you did, using manipulatives to solve the task) indicate that the task has a certain level of cognitive demand?
2. Is there a difference between "level of cognitive demand" and "difficulty"?
3. What effect does context (e.g., setting in which the task is used, students' prior experience, grade level) have on the level of cognitive demand required by a task?

Answers to the Task-Sorting Activity

"Answers" to the task-sorting activity may be found on the matrices that accompany the tasks.

Tasks categorized as “memorization” and “procedures without connections” are considered to be low level; tasks categorized as “procedures with connections” and “doing mathematics” are considered to be high level. Please see Stein and Smith (1998) for an extended discussion of these four levels of cognitive demand. Note that these answers are for the use of the facilitator in orchestrating the discussion. The goal of this activity is for teachers to participate in a thoughtful analysis of the tasks, not to come to a consensus about the placement of tasks. Also note that the characteristics of the tasks have been identified so as to facilitate the identification of counterexamples (e.g., if a teacher claims that manipulatives are only used in high-level tasks, you can use the matrix to identify a low-level task that also uses manipulatives).

Further Analysis of Tasks

You may want to have teachers read Smith and Stein (1998) and to differentiate further among tasks using their four categories (i.e., memorization, procedures without connections, procedures with connections, doing mathematics). Teachers could then use these four levels of cognitive demand (Smith and Stein 1998, p. 348) when considering aspects of their classroom practices. The following are suggestions that extend the thinking of the teachers and make connection to their classroom practices.

- Have the teachers collect all the tasks they use in math class for one week. Teachers can then analyze those tasks with regard to the levels of cognitive demand to investigate the level of cognitive demand required of the tasks they use on a daily basis.
- Have teachers analyze their current mathematics text with regard to levels of cognitive demand required by the tasks their students do on a daily basis.
- Have the teachers rewrite tasks that require a low level of cognitive demand to make them into tasks that require a high level of cognitive demand.

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Categorizing Mathematical Tasks

(Indicate whether each task is low or high level by placing an X in the appropriate column.)

TASK	LOW LEVEL	HIGH LEVEL
A		
B		
C		
D		
E		
F		
G		
H		
I		
J		
K		
L		
M		
N		
O		
P		

Martha's Carpeting Task

Martha was recarpeting her bedroom, which was 15 feet long and 10 feet wide. How many square feet of carpeting will she need to purchase?

Source: Stein et al. 2000.

Fencing Task

Ms. Brown's class will raise rabbits for their spring science fair. They have 24 feet of fencing with which to build a rectangular rabbit pen to keep the rabbits.

1. If Ms. Brown's students want their rabbits to have as much room as possible, how long would each of the sides of the pen be?
2. How long would each of the sides of the pen be if they had only 16 feet of fencing?
3. How would you go about determining the pen with the most room for any amount of fencing? Organize your work so that someone else who reads it will understand it.

Source: Stein et al. 2000.